BARI-TH/93-163
DSF-T-93/45
INFN-NA-IV-93/45
November 1993

DEPENDENCE OF THE FORM FACTORS OF $B \to \pi \ell \nu$ ON THE HEAVY QUARK MASS

Pietro Colangelo [1]

Istituto Nazionale di Fisica Nucleare, Sezione di Bari, Italy

and

Pietro Santorelli [2]

Istituto Nazionale di Fisica Nucleare, Sezione di Napoli, Italy

ABSTRACT

We use QCD sum rules to analyze the semileptonic transition $B \to \pi \ell \nu$ in the limit $m_b \to \infty$. We derive the dependence of the form factor $F_1(0)$ on the heavy quark mass, which is compatible with the expected dependence for the simple pole model of $F_1(q^2)$.

[1] E-mail address: COLANGELO@BARI.INFN.IT
[2] E-mail address: SANTORELLI@NA.INFN.IT

Heavy quark and chiral symmetries constrain the semileptonic decay $B \to \pi\ell\nu$ in the kinematical point where the pion is at rest in the rest frame of the decaying B meson (zero-recoil point). As a matter of fact, the form factors which parametrize the hadronic matrix element governing $B \to \pi\ell\nu$:

$$\langle\pi(p')|\, V_\mu \,|B(p)\rangle \;=\; F_1(q^2)\;(p+p')_\mu + \;\frac{m_B^2 - m_\pi^2}{q^2} q_\mu \;[F_0(q^2) - F_1(q^2)] \tag{1}$$

($q = p - p'$) can be written, near the zero-recoil point $q^2_{max} = (m_B - m_\pi)^2$, as follows [1, 2, 3, 4, 5, 6, 7] [3]:

$$F_1(q^2)\Big|_{q^2 \approx q^2_{max}} = \frac{f_{B^*}}{f_\pi}\,\frac{g_{B^*B\pi}}{1 - q^2/m^2_{B^*}} \tag{2}$$

and

$$F_0(q^2_{max}) = \frac{f_B}{f_\pi}\ . \tag{3}$$

In eq. (2) the dominance of the pole of the B^* meson, which is degenerate with the B meson in the limit $m_b \to \infty$, has been exploited; f_{B^*} and f_B are B^* and B meson leptonic constants, respectively; $g_{B^*B\pi}$ is the strong $B^*B\pi$ coupling constant.

The phenomenological importance of eqs. (2,3) is immediate in the light of the measurement of V_{ub}: since, neglecting the charged lepton mass,

$$\frac{d\Gamma(B \to \pi\ell\nu)}{dq^2} = \frac{G_F^2}{24\pi^3}|V_{ub}|^2|F_1(q^2)|^2|\vec{p}\,'_\pi(q^2)|^3 \tag{4}$$

where $\vec{p}\,'_\pi(q^2)$ is the pion three-momentum in the B rest frame at fixed q^2, one could compare the differential rates of $B \to \pi\ell\nu$ and $D \to \pi\ell\nu$ at the same (small) value of $\vec{p}\,'_\pi$; the ratio between the rates

$$\frac{d\Gamma(B \to \pi\ell\nu)/dq^2}{d\Gamma(D \to \pi\ell\nu)/dq^2}\Bigg|_{same\ \vec{p}\,'_\pi} = \frac{|V_{ub}|^2}{|V_{cd}|^2}\frac{|F_1^{B\to\pi}|^2}{|F_1^{D\to\pi}|^2}\Bigg|_{same\ \vec{p}\,'_\pi} \tag{5}$$

is given in terms of f_{B^*}/f_{D^*} and $g_{B^*B\pi}/g_{D^*D\pi}$ which can be measured and/or estimated by several methods [4], so that a measurement of the left-hand side of eq. (5) can provide a value of $|V_{ub}|$ with a procedure where the model dependence is drastically reduced.

This program, proposed in ref.[6], finds a relevant difficulty in the severe phase space suppression $|\vec{p}\,'_\pi(q^2)|^3$ in (4). For this reason it could be useful to investigate the dependence (if any) of the form factor $F_1(q^2)$ on the mass of the

[3] For a review see [8].

[4] We shall not discuss here the role of the breaking terms either of the chiral symmetry or of the heavy quark flavor symmetry.

heavy meson at fixed q^2, also far from q^2_{max}, i.e. in kinematical configurations where not only m_b, but also the momentum of the emitted pion represent heavy scale parameters. The aim is to attempt an extrapolation from $D \to \pi$ (when accurate experimental data will be available) to $B \to \pi$.

As we shall show below, such dependence can be predicted by relativistic QCD sum rules [9] at $q^2 = 0$.

The form factor F_1 in (1) has been studied by three-point function QCD sum rules, for a finite value of the b-quark mass, by a number of authors, adopting an analysis that can be applied also to the transitions $D \to (K, \pi)\ell\nu$ [10, 11, 12, 13] [5]. By studying the three-point correlator of a pseudoscalar current having the same quantum numbers of the B meson, of an axial current interpolating the pion, and of the flavor-changing current V_μ in (1), the following Borel improved sum rule can be derived:

$$\begin{aligned} & f_\pi f_B \frac{m_B^2}{m_b} F_1(q^2=0) \exp\left\{-\frac{m_B^2}{M^2} - \frac{m_\pi^2}{M'^2}\right\} \\ = & \frac{1}{(2\pi)^2} \int_D ds\, ds' \rho(s,s') \; \exp\left\{-\frac{s}{M^2} - \frac{s'}{M'^2}\right\} \\ - & \frac{<\bar{q}q>}{2} \; \exp\left\{-\frac{m_b^2}{M^2}\right\} \left[1 - \frac{m_0^2}{6}\left(\frac{3m_b^2}{2M^4} - \frac{2}{M^2}\right)\right] \end{aligned} \tag{6}$$

where the integration region D is $m_b^2 \le s \le s_0,\ 0 \le s' \le min(s'_0, s - m_b^2)$ (s_0, s'_0 are effective thresholds separating the resonance region from the continuum); the perturbative spectral function ρ, at the lowest order in α_s, is given by:

$$\begin{aligned} \rho(s,s') = & \frac{3m_b}{2(s-s')^3}\Big[2\tilde{\Delta}(u-s') + s'(u-4s) - \frac{2m_b}{(s-s')}\Big(\tilde{\Delta}^2(u^2 - 3us' + 2ss') \\ + & 2\tilde{\Delta}s'(u^2 - 3us + 2ss') + 3ss'^2(2s-u)\Big)\Big] \end{aligned} \tag{7}$$

($\tilde{\Delta} = s - m_b^2$, $u = s + s'$); M, M' are the Borel parameters associated to B and to the pion channel, respectively; $<\bar{q}q>$ is the condensate of dimension 3, whereas m_0^2 is connected to the condensate of dimension 5: $m_0^2 = <\bar{q}g\sigma Gq> / <\bar{q}q>$. The mass of the light quarks has been neglected.

The limit $m_b \to \infty$ can be performed by changing the variables associated to the heavy quark channel in terms of low-energy variables. Using: $s = m_b^2 + 2m_b\omega$, $s_0 = m_b^2 + 2m_b\omega_0$, $M^2 = m_b T$ and $m_B = m_b + \Lambda/2$, i.e. using the same procedure adopted in [15] in the analysis of the semileptonic transition $(\bar{q}Q) \to (\bar{q}Q')\ \ell\nu$, eq.(6) can be written as follows:

[5] For a review see [14].

$$
\begin{aligned}
& f_\pi(\sqrt{m_b} f_B)\left(1+\frac{\Lambda}{m_b}\right)(\sqrt{m_b} F_1(0)) \exp\left\{-\frac{\Lambda}{T}-\frac{m_\pi^2}{M'^2}\right\} \\
= & \frac{3}{\pi^2 m_b}\int_0^{\omega_0} d\omega \int_0^{s'_0} ds' \omega \exp\left\{-\frac{2\omega}{T}-\frac{s'}{M'^2}\right\}-\frac{<\bar{q}q>}{2}\left[1-\frac{m_0^2}{4T^2}\right] \quad . \quad (8)
\end{aligned}
$$

From this equation the behavior of $F_1(0)$ versus m_b can be derived. In fact, Λ, which is related to the binding energy of the heavy-light quark system, remains finite in the infinite heavy quark mass limit; therefore, since the scaling law for f_B, for $m_b \to \infty$, is $1/\sqrt{m_b}$ modulo logs, one obtains that also $F_1(0)$ scales as $1/\sqrt{m_b}$: this is a consequence of the fact that the perturbative term is subleading in the heavy quark limit, the sum rule being determined by the non perturbative $D=3$ and $D=5$ contributions. A similar result has been found for the form factors V and A_1 governing $B \to \rho\ell\nu$ and for the form factor of the rare $B \to K^*\gamma$ decay [16]. It is worth observing that the scaling law $1/\sqrt{m_b}$ for F_1 at $q^2=0$ is compatible with the correct dependence of F_1 on m_b at q^2_{max} in eq. (2):

$$
F_1(q^2_{max}) = \frac{f_{B^*}\ m_{B^*}\ g_{B^*B\pi}}{2\ f_\pi\ (\delta_B + m_\pi)} \simeq \frac{\sqrt{m_b}}{\delta_B + m_\pi} \tag{9}
$$

(with $\delta_B = m_{B^*} - m_B = \mathcal{O}(1/m_b)$) and with a simple pole evolution from q^2_{max} to $q^2=0$. We shall discuss this point at the end of the paper; here we want to show that the variables adopted above are true low energy variables for the system we are considering.

In order to do that, let us analyze the form factors of $B \to \pi\ell\nu$ in the heavy quark effective theory (HQET) [17]. As discussed in ref.[6], the matrix element in (1) can be written, in the framework of HQET, in terms of the B meson velocity: $v = p/m_B$, and of the energy of the emitted pion in the B rest frame:

$$
v \cdot p' = \frac{m_B^2 + m_\pi^2 - q^2}{2m_B} \quad . \tag{10}
$$

Accordingly, eq.(1) can be rewritten as follows:

$$
\frac{\langle \pi(p')|\, V_\mu\, |B(v)\rangle}{\sqrt{m_B}} \ =\ 2\ f_1(v\cdot p')\ v_\mu\ +\ 2\ f_2(v\cdot p')\ \frac{p'_\mu}{(v\cdot p')}\ , \tag{11}
$$

where the functions f_1 and f_2 are universal, in the sense that they become independent of the heavy quark mass m_b in the limit $m_b \to \infty$, for values of $v \cdot p'$ which do not scale as m_b. It is immediate to derive the relations between the form factors in (1) and (11):

$$
F_1(q^2) \ = \ \sqrt{m_B}\left\{\frac{f_1(v\cdot p')}{m_B} + \frac{f_2(v\cdot p')}{(v\cdot p')}\right\} \tag{12}
$$

$$F_0(q^2) = \frac{2m_B^{3/2}}{m_B^2 - m_\pi^2} \times \times \left\{ [f_1(v\cdot p') + f_2(v\cdot p')] - \frac{(v\cdot p')}{m_B} \left[f_1(v\cdot p') + \frac{m_\pi^2}{(v\cdot p')^2} f_2(v\cdot p') \right] \right\} . \tag{13}$$

QCD sum rules can be employed in the evaluation of f_1 and f_2; the approach is similar to that used in the determination of the Isgur-Wise function [15], with the difference that, in this case, the effective theory is only applied to B-meson channel. The starting point is the correlator

$$T_{\mu\nu}(k, p', v\cdot p') = i^2 \int dx\, dy\, e^{i(p'\cdot x - k\cdot y)} \; < 0|T\{j_\nu(x)\hat{V}_\mu(0)\hat{J}_5^\dagger(y)\}|0> \; ; \tag{14}$$

the axial current $j_\nu(x)$ interpolates the pion; the other currents are $\hat{J}_5(y) = \bar{q}(y)i\gamma_5 h_v(y)$ and $\hat{V}_\mu(0) = \bar{q}(0)\gamma_\mu h_v(0)$, where h_v is the velocity dependent $b-$quark field in the effective theory, whose residual "off-shell" momentum is k. The matrix element of $\hat{J}_5$ between the vacuum state and the B-meson state defines the scale-dependent universal leptonic constant $\hat{F}(\mu)$:

$$< 0|\hat{J}_5|B(v) >= \hat{F}(\mu) \; , \tag{15}$$

where μ is the renormalization scale, and the connection of $\hat{F}$ to the $B-$meson leptonic constant f_B is given by

$$f_B\sqrt{m_B} = C_1(\mu)\hat{F}(\mu) + O(1/m_b) \; , \tag{16}$$

$C_1(\mu)$ being a Wilson coefficient.

By applying to the correlator (14) the usual techniques of the QCD sum rules method, the following Borel improved rules for $f_1(v\cdot p')$ and $f_2(v\cdot p')$ can be worked out:

$$\begin{aligned} & 2\, f_\pi\, \hat{F}\, f_1(v\cdot p')\, \exp\left\{-\frac{\Lambda}{T} - \frac{m_\pi^2}{M'^2}\right\} = \\ = & \frac{1}{(2\pi)^2}\int_0^{\omega_0} d\omega \int_0^{\hat{s}'} ds'\, \rho_1(\omega, s', v\cdot p')\, \exp\left\{-\frac{\omega}{T} - \frac{s'}{M'^2}\right\} \\ - & < \bar{q}q > \left[1 - m_0^2\left(\frac{1}{4T^2} + \frac{2}{3}\frac{(v\cdot p')}{TM'^2}\right)\right] \end{aligned} \tag{17}$$

and

$$\begin{aligned} & \frac{2\, f_\pi\, \hat{F}}{(v\cdot p')} f_2(v\cdot p')\, \exp\left\{-\frac{\Lambda}{T} - \frac{m_\pi^2}{M'^2}\right\} = \\ = & \frac{1}{(2\pi)^2}\int_0^{\omega_0} d\omega \int_0^{\hat{s}'} ds'\, \rho_2(\omega, s', v\cdot p')\, \exp\left\{-\frac{\omega}{T} - \frac{s'}{M'^2}\right\} \\ - & \frac{m_0^2 < \bar{q}q >}{3TM'^2} \; , \end{aligned} \tag{18}$$

with the spectral functions ρ_1 and ρ_2 given by:

$$\rho_1(\omega, s', v{\cdot}p') \; = -\frac{3}{8}\,\frac{s'}{\Delta^{\frac{5}{2}}}\left\{10\omega(v\cdot p')^2 + 2\omega s' - 4(v\cdot p')^3 - 8(v\cdot p')s' - 3\omega^2(v\cdot p')\right\} \tag{19}$$

and

$$\begin{aligned}\rho_2(\omega, s', v\cdot p') \;\;&=\;\; -\frac{3}{4}\,\frac{s'}{\Delta^{\frac{5}{2}}}\{8s'(v\cdot p')^2 + 4s'^2 - 4\omega(v\cdot p')^3 \\ &-\;\; 8\omega(v\cdot p')s' + 2\omega^2(v\cdot p')^2 + \omega^2 s'\} \;\; ;\end{aligned} \tag{20}$$

ω_0 is the effective threshold in the heavy quark channel and the upper integration limit in s' in (17,18) is $\hat{s}' = min(s'_0, [(v\cdot p') - \omega]^2)$. The factor Δ is given by $\Delta = (v\cdot p')^2 - s'$. It should be noticed the appearance of a branch point in the perturbative spectral functions ρ_1 and ρ_2, due to the factor $\Delta^{-\frac{5}{2}}$, for small values of $v\cdot p'$, implying that the rules cannot describe a process with small pion energy. This is not unexpected, since the euclidean region in the $t-$channel, where the QCD perturbative calculation of the spectral functions can be performed, extends towards large values $-q^2$, i.e. large values of $v{\cdot}p'$: we shall extrapolate the analysis of the sum rules to values of $v\cdot p'$ of the order of 1 GeV, being aware that the accuracy of the prediction becomes poor in this range of pion energy.

The last point to be mentioned, before presenting the numerical analysis of eqs.(17, 18), is that also f_1 and f_2, obtained using the effective field $h_v(x)$, depend on the subtraction point μ; however, in the following we neglect this dependence since we choose to work at the lowest order in α_s.

In fig.1 we present the form factors f_1 and f_2 obtained from eqs.(17, 18) using $s'_0 = 0.8\ GeV^2$, and the set of low energy parameters fixed in ref. [15] by the analysis of the leptonic constant $\hat{F}$ in (15): $\omega_0 = 2.1\ GeV$, $\Lambda = 1\ GeV$ and $\hat{F} = 0.47\ GeV^{3/2}$, or $\omega_0 = 2.5\ GeV$, $\Lambda = 1.25\ GeV$ and $\hat{F} = 0.58\ GeV^{3/2}$. We fix the values of the Borel parameters: $T = 3\ GeV$ and $M'^2 = 3\ GeV^2$ after having checked the existence of a duality region around these values for any $v\cdot p'$.

The rapid change in the behaviour of $f_2(v\cdot p')$ for $v\cdot p' \leq 1.3\ GeV$ can be ascribed to the presence of the anomalous threshold discussed above.

Using eqs.(12,13) it is possible to reconstruct $F_1(q^2)$ and $F_0(q^2)$ for intermediate values of q^2; the result is depicted in fig.2, where the form factors are displayed up to $q^2 = 0$. The obtained F_1 is compatible with the outcome of the finite mass calculation [12]: it rapidly increases, and can be fitted with the simple pole form: $F_1(q^2) = F_1(0)/(1 - q^2/m^2_{pole})$, with $F_1(0) = 0.24$ and $m_{pole} = 5.51\ GeV$. On the other hand, F_0 is nearly constant in q^2.

It is interesting to notice that the combination of f_1 and f_2 which reconstructs F_1 reproduces eq.(8), i.e. gives a leading non perturbative term and a subleading perturbative contribution. A warning is in order, however. The functions $f_1(v{\cdot}p')$ and $f_2(v\cdot p')$ obtained by (17, 18) are the leading terms of an expansion in $1/m_b$ [6]; in this expansion, the pion energy $v\cdot p'$ should be kept fixed, i.e. cannot scale

as m_b (as it happens at $q^2 = 0$) since in this case the neglected contributions are formally of the the same order of the terms we are considering. On the other hand, the procedure we have followed reproduces eq.(8) and is in numerical agreement with the outcome of the calculation made at finite m_b [12]: this suggests that the additional contributions in the $1/m_b$ expansion that reconstruct the form factors should be small also at $q^2 = 0$.

Let us come back to the analytic dependence of $F_1(0)$ on m_b. As mentioned above, the scaling law $1/\sqrt{m_b}$ obtained from eq.(8) is in agreement with a polar dependence in the range from q^2_{max} to $q^2 = 0$, with the pole given by the B^* resonance. This prediction suggests that possible non polar components of the form factor near q^2_{max}, discussed in [7, 18], are extended in a limited range of q^2 or they scale as $1/\sqrt{m_b}$ or faster when $m_b \to \infty$.

Other models predict the dependence of $F_1(0)$ on m_b. In the BSW constituent quark model [19] the form factors at $q^2 = 0$ are obtained by computing an overlap of mesonic wave functions which are solutions of a relativistic scalar harmonic oscillator potential; the evolution in q^2 is assumed to be a simple pole. A direct inspection of the m_b dependence of $F_1(0)$ in this model gives $F_1(0) \simeq m_b^{-3/2}$, a behavior incompatible, as already observed in [18], with the scaling law of $F_1(q^2_{max})$ predicted in (1,9) and with the assumption of a simple pole dependence of the form factor. The same inconsistency is present in similar models [20].

The dependence of $F_1(0)$ on m_b can also be predicted by light-cone QCD sum rules [21, 22]. This method relies on the possibility of computing the two-point correlator of a quark current interpolating the B-meson and of the flavour-changing current in (1) calculated between the vacuum state and the light meson state (π). Also in this case the predicted dependence for $F_1(0)$ is $m_b^{-3/2}$ and therefore the scaling law is incompatible with the correct behaviour at q^2_{max} and with the claim [22] that the observed q^2 dependence of the form factor is polar. The origin of such inconsistency is currently under investigation.

Let us conclude with a comment on the results of the present study. The measurement of V_{ub}, which is of prime importance in the phenomenology of the Standard Model, cannot be performed by the simple observation of the decay $B \to \pi \ell \nu$ without referring to models for the evaluation of the hadronic matrix element. Here we have obtained indications that F_1 is polar in q^2, and that the scaling law $F_1(0) \simeq 1/\sqrt{m_b}$ should be fulfilled, at least asymptotically in the heavy quark mass. This information allows to use the full sample of pions from semileptonic B decays; V_{ub} could be determined by comparing eq.(4) with the analogous expression for $D \to \pi \ell \nu$ and using the experimental result of this last decay. The only drawback, as usual in these considerations, is represented by the size of $1/m_Q$ corrections, an argument which deserves an independent investigation.

Acknowledgments

We thank P.Ball, F.Buccella, G.Nardulli and N.Paver for fruitful discussions.

References

[1] M.B.Wise, Phys. Rev. D 45 (1992) 2188

[2] G.Burdman and J.F.Donoghue, Phys. Lett. B 280 (1992) 287

[3] L.Wolfenstein, Phys. Lett. B 291 (1992) 447

[4] R.Casalbuoni, A.De Andrea, N.Di Bartolomeo, G.Feruglio, R.Gatto and G. Nardulli, Phys. Lett. B 299 (1993) 139.

[5] N.Isgur and M.B.Wise, Phys. Rev. D41 (1990) 151.

[6] G.Burdman, Z.Ligeti, M.Neubert and Y.Nir, Report SLAC-PUB-6345, WIS-93/86/Sep-PH (September 1993).

[7] N.Paver and Riazuddin , Report IC/93/221, UTS-DFT-93-20 (August 1993).

[8] M.B.Wise, Report CALT-68-1860.

[9] M.A.Shifman, A.I.Vainshtein and V.I.Zakharov, Nucl. Phys. B 147, 385 (1979). For a review see the reprints volume: "Vacuum structure and QCD Sum Rules", M.A.Shifman, *ed.*, North-Holland, 1992.

[10] T.M.Aliev, V.L.Eletsky and Ya.I.Kogan, Sov. J. Nucl. Phys. 40 (1984) 527; T.M.Aliev, A.A.Ovchinnikov and V.A.Slobodenyuk, report IC/89/382 (1989) (unpublished); A.A.Ovchinnikov, Sov. J. Nucl. Phys. 50 (1989) 519.

[11] P.Ball, V.M.Braun, H.G.Dosch and M.Neubert, Phys. Lett. B259 (1991) 481; P.Ball, V.M.Braun and H.G.Dosch, Phys. Rev. D44 (1991) 3567; P.Ball, V.M.Braun and H.G.Dosch, Phys. Lett. B273 (1991) 316.

[12] P.Ball, Phys. Rev. D 48 (1993) 3190.

[13] S.Narison, Phys. Lett. B 283 (1992) 384.

[14] P.Colangelo, Report BARI-TH/93-152 (September 1993).

[15] M.Neubert, Phys. Rev. D 45 (1992) 2451.

[16] P.Colangelo, C.A.Dominguez, G.Nardulli and N.Paver, Phys. Lett. B 317 (1993) 183.

[17] N. Isgur and M.B. Wise, Phys. Lett. B 232,1989,113,
N. Isgur and M.B. Wise, Phys. Lett. B 237,1990,527,
H. Georgi, Phys. Lett. B 240,1990,447.

[18] G.Burdman and J.F.Donoghue, Phys. Rev. Lett. 68 (1992) 2887.

[19] M.Wirbel, B.Stech and M.Bauer, Zeit. fur Phys. C - Particles and Fields 29 (1985) 637.

[20] J.G.Korner and G.A.Schuler, Zeit. fur Phys. C - Particles and Fields 38 (1988) 511.

[21] V.L.Chernyak and I.R.Zhitnitsky, Nucl. Phys. B345 (1990) 137.

[22] V.M.Belyaev, A.Khodjamirian and R.Ruckl, Zeit. fur Phys. C 60 (1993) 349.

Figure Captions

Fig 1: The form factors $f_1(v \cdot p')$ and $f_2(v \cdot p')$.

Fig 2: The form factors $F_1(q^2)$ and $F_0(q^2)$.